Infinity Masterclasses
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MTSE 2026 | CLASS 7–8

Detailed Solutions

Step-by-step solutions



Answer Key

Q1 **A**	Q2 **B**	Q3 **C**	Q4 **D**	Q5 **B**
Q6 **C**	Q7 **A**	Q8 **C**	Q9 **D**	Q10 **B**
Q11 **D**	Q12 **D**	Q13 **B**	Q14 **D**	Q15 **B**
Q16 **A**	Q17 **A**	Q18 **D**	Q19 **C**	Q20 **A**
Q21 **B**	Q22 **A**	Q23 **C**	Q24 **C**	Q25 **B**

Colours follow the case studies: 1 Polygon Detective · 2 Product Between Two Squares · 3 Repeating Pattern · 4 Equal Share · 5 City Grid · 6 Matchmaker's Count

How to use this booklet. Each solution has a short title, numbered steps and a coloured answer box that shows the correct option exactly as printed in the paper. A "Watch out" note names the mistake behind a tempting wrong option.

Scoring. 25 questions × 4 marks = 100 marks. There is no negative marking. Correct options: A × 6, B × 7, C × 6, D × 6.

CASE STUDY 1

The Polygon Detective's Fan



QUESTIONS
1 – 5
5 × 4 = 20 marks

1 Sides from the angle sum

The inside angles of a convex polygon add up to 1980°. How many sides does it have?

Step 1. Use the rule: inside-angle sum = $(n - 2) \times 180°$, where n is the number of sides.

$$(n - 2) \times 180 = 1980$$

Step 2. Divide both sides by 180.

$$n - 2 = 1980 \div 180 = 11$$

Step 3. Add 2.

$$n = 13$$

Step 4. Check: 11 triangles × 180° = 1980°. So the polygon has 13 sides.

ANSWER A **13**

2 Diagonals from one corner

From one corner of a convex polygon, 9 diagonals are drawn. What is the total of the inside angles?

Step 1. The diagonals from one corner cut the polygon into triangles. 9 diagonals make 9 + 1 = 10 triangles.

Step 2. Each triangle has an angle sum of 180°.

$$10 \times 180° = 1{,}800°$$

Step 3. Check: 10 triangles means $n - 2 = 10$, so the polygon has 12 sides. A corner of a 12-sided polygon has 12 − 3 = 9 diagonals. This matches.

Watch out. *1,620° (9 × 180°) forgets the extra triangle. Always add 1 to the number of diagonals.*

ANSWER B **1,800°**

3 A regular polygon

Each inside angle of a regular polygon measures 156°. How many sides does it have?

Step 1. A regular polygon has all sides equal and all angles equal. So each inside angle is the sum divided by n.

$$(n - 2) \times 180 \div n = 156$$

Step 2. Multiply both sides by n.

$$180n - 360 = 156n$$

Step 3. Collect the n terms.

$$24n = 360, \text{ so } n = 15$$

Step 4. Check: 13 × 180° = 2,340°, and 2,340° ÷ 15 = 156°.

Step 5. Quick way: each outside angle is 180° − 156° = 24°. All the outside angles of a convex polygon add up to 360°, and 360° ÷ 24° = 15.

ANSWER C **15**

4 One angle is different

A convex polygon has n sides. One inside angle is 100°. Each of the other inside angles is 140°.

Step 1. Count the total in two ways. One angle is 100°. The other $(n - 1)$ angles are 140° each.

$$\text{Total} = 100 + 140(n - 1)$$

Step 2. The total must also equal $(n - 2) \times 180$.

$$100 + 140n - 140 = 180n - 360$$

Step 3. Simplify the left side and collect terms.

$$140n - 40 = 180n - 360, \text{ so } 320 = 40n$$

$$n = 8$$

Step 4. Check: 100° + 7 × 140° = 1,080° and (8 − 2) × 180° = 1,080°.

ANSWER D 8

5 Two polygons

Polygon A has k sides. Polygon B has (k + 3) sides. B's angle sum is double A's. How many triangles does B make from one corner?

Step 1. A's angle sum is $(k - 2) \times 180°$. B's angle sum is $(k + 3 - 2) \times 180° = (k + 1) \times 180°$.

Step 2. B is double A. The 180° cancels.

$$k + 1 = 2(k - 2) = 2k - 4, \text{ so } k = 5$$

Step 3. So A is a pentagon and B has 5 + 3 = 8 sides.

Step 4. From one corner, a polygon with 8 sides is cut into 8 − 2 = 6 triangles.

Step 5. Check: pentagon 540°, octagon 1,080°, and 1,080 = 2 × 540.

Watch out. *The question asks for the number of triangles, not the number of sides or k.*

ANSWER B 6

CASE STUDY 2

The Product Hidden Between Two Squares



QUESTIONS
6 – 10
5 × 4 = 20 marks

6 Use the midpoint

Find 1,997 × 2,003.

Step 1. Find the midpoint of 1,997 and 2,003. It is 2,000. Each number is 3 away from it: 2,000 − 3 and 2,000 + 3.

Step 2. Use $(n - d)(n + d) = n^2 - d^2$ with $n = 2{,}000$ and $d = 3$.

$$2{,}000^2 - 3^2 = 4{,}000{,}000 - 9 = 3{,}999{,}991$$

ANSWER C 3,999,991

7 Sum and difference

Two numbers add up to 160 and differ by 6. What is their product?

Step 1. The midpoint of the two numbers is half the sum: 160 ÷ 2 = 80.

Step 2. They differ by 6, so each is 3 away from 80. The numbers are 77 and 83.

Step 3. Use $n^2 - d^2$ with $n = 80$ and $d = 3$.

$$80^2 - 3^2 = 6{,}400 - 9 = 6{,}391$$

Step 4. Check: 77 × 83 = 6,391.

Watch out. *6,400 is only n^2. Remember to subtract d^2.*

ANSWER A 6,391

8 Two products compared

Find 20,007 × 19,993 − 20,010 × 19,990.

Step 1. Both products have the midpoint 20,000.

$$20{,}007 \times 19{,}993 = 20{,}000^2 - 7^2$$
$$20{,}010 \times 19{,}990 = 20{,}000^2 - 10^2$$

Step 2. Subtract. The big $20{,}000^2$ terms cancel, so there is no need to calculate them.

$$(20{,}000^2 - 49) - (20{,}000^2 - 100) = -49 + 100 = 51$$

Watch out. *The answer −51 comes from subtracting the wrong way round. The first product has the smaller d, so it is the larger number.*

ANSWER C **51**

9 Difference and product known

Two positive whole numbers differ by 6. Their product is 2,295. What is their sum?

Step 1. Let the midpoint be n. The numbers are $n - 3$ and $n + 3$, because they differ by 6.

$$n^2 - 3^2 = 2{,}295, \text{ so } n^2 = 2{,}304$$

Step 2. Find the number whose square is 2,304. It is between 40 and 50 ($40^2 = 1{,}600$ and $50^2 = 2{,}500$). The last digit 4 means the number ends in 2 or 8. So $n = 48$.

Step 3. The numbers are 45 and 51. Check: $45 \times 51 = 2{,}295$.

Step 4. The sum is $45 + 51 = 96$. This is $2n = 2 \times 48$.

ANSWER D **96**

10 A long product

Find $(1 - 1/2^2)(1 - 1/3^2)(1 - 1/4^2) \ldots (1 - 1/10^2)$.

Step 1. Use the hint on every bracket: $1 - 1/k^2 = (1 - 1/k)(1 + 1/k)$.

Step 2. Write $1 - 1/k = (k - 1)/k$ and $1 + 1/k = (k + 1)/k$. Make two products, one for each kind of bracket.

Step 3. The first product. Each top number cancels the bottom number before it. Only 1 on the top and 10 on the bottom are left.

$$\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \ldots \times \frac{9}{10} = \frac{1}{10}$$

Step 4. The second product. Each bottom number cancels the top number before it. Only 11 on the top and 2 on the bottom are left.

$$\frac{3}{2} \times \frac{4}{3} \times \frac{5}{4} \times \ldots \times \frac{11}{10} = \frac{11}{2}$$

Step 5. Multiply the two results.

$$\frac{1}{10} \times \frac{11}{2} = \frac{11}{20}$$

ANSWER B **11/20**

CASE STUDY 3

The Repeating Pattern Detective



QUESTIONS
11 – 15
5 × 4 = 20 marks

11 Beads in a cycle

Red, Green, Blue, Yellow repeat. What colour is the 50th bead?

Step 1. One cycle has 4 beads. Divide 50 by 4.

$$50 \div 4 = 12 \text{ remainder } 2$$

Step 2. 12 full cycles use 48 beads. The 50th bead is the 2nd bead of the next cycle.

Step 3. The 2nd bead in the cycle is Green.

Watch out. *If the remainder is 0, the bead is the last one in the cycle, not the first.*

ANSWER D **Green**

12 Digits of 1/7

What is the 2,026th digit after the decimal point of 1/7 = 0.142857142857...?

Step 1. The digits 142857 repeat. One cycle has 6 digits.

Step 2. Divide 2,026 by 6.

$$2{,}026 \div 6 = 337 \text{ remainder } 4$$

Step 3. 337 full cycles use 2,022 digits. The 2,026th digit is the 4th digit of the cycle.

Position in cycle	1	2	3	4	5	6
Digit	1	4	2	8	5	7

Step 4. The 4th digit is 8.

ANSWER D 8

13 The month with five Fridays, Saturdays and Sundays

A 31-day month has exactly five Fridays, five Saturdays and five Sundays. On which day does it begin?

Step 1. 31 days = 4 weeks + 3 extra days. 4 weeks give 4 of every day.

Step 2. The 3 extra days are the first three days of the month, so these days appear 5 times.

Step 3. The days that appear 5 times are Friday, Saturday and Sunday, in that order.

Step 4. So the first three days of the month are Friday, Saturday and Sunday. The month begins on a Friday.

ANSWER B Friday

14 Sum of a repeating list

3, 7, 1, 5 repeat forever. What is the sum of the first 50 numbers?

Step 1. One cycle is 3, 7, 1, 5. Its sum is 3 + 7 + 1 + 5 = 16.

Step 2. Divide 50 by 4.

$$50 \div 4 = 12 \text{ remainder } 2$$

Step 3. 12 full cycles give 12 × 16 = 192.

Step 4. The last 2 numbers are 3 and 7. Their sum is 10.

$$192 + 10 = 202$$

Watch out. *208 would be 13 full cycles. There are only 2 numbers left after 12 cycles.*

ANSWER D 202

15 Last digit of a sum of powers

What is the last digit of $2^{2026} + 3^{2026}$?

Step 1. Look at the last digits of the powers. They repeat in a cycle of 4.

Power	1st	2nd	3rd	4th
Last digit of 2^n	2	4	8	6
Last digit of 3^n	3	9	7	1

Step 2. Divide the exponent by 4.

$$2026 \div 4 = 506 \text{ remainder } 2$$

Step 3. So both powers end like the 2nd power in their cycle: 2^{2026} ends in 4 and 3^{2026} ends in 9.

Step 4. Add the last digits.

$$4 + 9 = 13, \text{ so the last digit is } 3$$

ANSWER B 3

CASE STUDY 4

The Equal Share of a Set



QUESTIONS
16 – 17
2 × 4 = 8 marks

16 Remove one number

Five numbers have an average of 22. One number is 14. What is the average of the other four?

Step 1. Total of the five numbers = average × how many.

$$5 \times 22 = 110$$

Step 2. Remove 14 from the total.

$$110 - 14 = 96$$

Step 3. Four numbers are left. Share the total equally among them.

$$96 \div 4 = 24$$

Watch out. *The average goes up because the number removed (14) is smaller than the average (22).*

ANSWER A **24**

17 Replace a student

8 students have an average of 82. A student who scored 70 is replaced. The average rises to 85. What did the new student score?

Step 1. Old total = 8 × 82 = 656.

Step 2. New total = 8 × 85 = 680.

Step 3. The total rose by 680 − 656 = 24.

Step 4. The new student scored 24 more than the student who left.

$$70 + 24 = 94$$

Step 5. Check: 656 − 70 + 94 = 680 ✓.

Watch out. *The team stays at 8 students. Do not divide by 7.*

ANSWER A **94**

CASE STUDY 5

The City Grid



QUESTIONS
18 – 20
3 × 4 = 12 marks

18 Home to the Shop

Home (0, 0) to the Shop at (4, 2). How many different shortest routes?

Step 1. Use the counting rule. The number of routes to a circle = routes from the circle on its left + routes from the circle below it.

Step 2. The bottom row and the left column are all 1.

y = 2	1	3	6	10	15
y = 1	1	2	3	4	5
y = 0	1	1	1	1	1
	x = 0	x = 1	x = 2	x = 3	x = 4

Step 3. Each number is the sum of its left and lower neighbours. For example, 6 = 3 + 3 and 15 = 10 + 5.

Step 4. There are 15 shortest routes to (4, 2).

Step 5. Check another way: each route is 4 East and 2 North moves, 6 moves in all. Choose which 2 of the 6 moves are North: $6 \times 5 \div 2 = 15$.

ANSWER D 15

19 Stopping at the Post Office

Home to (4, 3), stopping at (2, 1). How many shortest routes?

Step 1. Split the trip into two parts: Home to (2, 1), then (2, 1) to (4, 3).

Step 2. Part 1: Home to (2, 1) is 2 East and 1 North. From the table in Q18, there are 3 routes.

Step 3. Part 2: (2, 1) to (4, 3) is 2 East and 2 North. Use the counting rule from (2, 1) as the new start.

2 North	1	3	6
1 North	1	2	3
start	1	1	1

Step 4. There are 6 routes for part 2.

Step 5. Every route for part 1 can be joined with every route for part 2. Multiply.

$$3 \times 6 = 18$$

ANSWER C 18

20 A closed intersection

Home to (4, 2). The intersection (2, 1) is closed. How many shortest routes?

Step 1. Use the counting rule again. Put 0 at the closed intersection, because no route can pass through it.

y = 2	1	3	3	4	6
y = 1	1	2	0	1	2
y = 0	1	1	1	1	1
	x = 0	x = 1	x = 2	x = 3	x = 4

Step 2. Check some numbers: (3, 1) = 0 + 1 = 1. (4, 1) = 1 + 1 = 2. (2, 2) = 3 + 0 = 3. (3, 2) = 3 + 1 = 4. (4, 2) = 4 + 2 = 6.

Step 3. Check another way: there are 15 routes without a closed road. Routes through (2, 1) = 3 × 3 = 9. So 15 − 9 = 6.

ANSWER A 6

CASE STUDY 6

The Matchmaker's Count



QUESTIONS **21 – 25**

5 × 4 = 20 marks

21 Seven players

Seven players play every other player once. How many matches?

Step 1. Let the players walk in one by one. Each new player plays all the players already there.

$$0 + 1 + 2 + 3 + 4 + 5 + 6 = 21$$

Step 2. Check another way: each of the 7 players plays 6 others, which makes $7 \times 6 = 42$. But every match was counted twice (once for each player). So $42 \div 2 = 21$.

ANSWER B 21

22 Find the number of players

There are 45 matches in all. How many players took part?

Step 1. Add 1 + 2 + 3 + … until the total reaches 45.

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45$$

Step 2. The last number added is 9. It is the number of matches played by the last player to walk in. That player plays the 9 players already there, so there are 10 players.

Step 3. Check: $n(n-1) \div 2 = 10 \times 9 \div 2 = 45$.

Watch out. *9 is the largest number added, not the number of players.*

ANSWER A 10

23 Two new players join

8 players have played. Then 2 new players join. How many more matches?

Step 1. Each new player plays the 8 players already there.

$$2 \times 8 = 16$$

Step 2. The two new players also play each other. That is 1 more match.

$$16 + 1 = 17$$

Step 3. Check: 10 players have 45 matches. 8 players have 28 matches. $45 - 28 = 17$.

Watch out. *16 forgets the match between the two new players.*

ANSWER C 17

24 Adding the records

10 players. Each writes down how many matches they played. The total is 90. How many matches were played?

Step 1. One match is played by 2 players. So each match appears in 2 records.

Step 2. So the total of the records is 2 times the number of matches.

$$\text{Matches} = 90 \div 2 = 45$$

Step 3. Check: each of the 10 players plays 9 matches, so the total of the records is $10 \times 9 = 90$ ✓.

Watch out. *90 is the total of the records, not the number of matches.*

ANSWER C 45

25 Fewest time slots

7 players. Everyone plays everyone once. A player plays only one game in a time slot. What is the fewest number of time slots?

Step 1. Total games = $7 \times 6 \div 2 = 21$.

Step 2. In one time slot, each game uses 2 players. 7 players make at most 3 games, and 1 player rests.

Step 3. So at most 3 games in a time slot.

$$21 \div 3 = 7 \text{ time slots at least}$$

Step 4. Now show that 7 time slots are enough. Call the players A to G. In each time slot, one player rests.

Slot	Games	Rests
1	BG, CF, DE	A
2	AB, CG, DF	E
3	AC, DG, EF	B
4	AD, BC, EG	F
5	AE, BD, FG	C
6	AF, BE, CD	G
7	AG, BF, CE	D

Step 5. All 21 games are played exactly once. So 7 time slots are enough.

Watch out. *6 is a tempting answer, because each player plays 6 games. But with 7 players, someone must rest in every slot.*

ANSWER B 7